Example 3.14. Location A for a factory is expected to produce an annual profit that is $10,000 per year greater than if location B is chosen. This additional profit is spread evenly over the entire year and constitutes a continuous flow of $10,000 per year. The profit is continuously reinvested at the rate of return of 14 percent per year. At the end of 8 years, what is the difference between the worths of the two investments?

Solution. Since the $10,000 per year difference is a continuous flow continuously compounded, the difference between future worths is

\[(\$10,000) e^{0.14(8)} - 1 \approx \$147,490\]

3.19 SUMMARY

There are several levels of economic analysis higher than that approached by this chapter. The complications in accounting, financing, and tax computations involve sophistications beyond those presented here. The stage achieved by this chapter might be described as the second level of economic analysis. The first level would be a trivial one of simply totaling costs with no consideration of the time value of money. The second level introduces the influences of interest, which imposes the dimension of time as well as amount in assessing the value of money.

The methods of investment analyses explained in this chapter are used repeatedly in engineering practice, and in most cases engineers are not required to go beyond these principles. These methods also are the base for extensions into more complex economic analyses.

PROBLEMS

3.1. Using a computer, calculate your personal set of tables for the factors f/p, f/a, p/a, and GPWF. Devote a separate page to each of the factors, label adequately, and calculate at the interest rates of 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 25 percent. Calculate for the following interest periods: 1 to 20 by ones, 22 to 30 by twos, and 30 to 60 by fives. Print out the factors to four places after the decimal point.

3.2. Annual investments are being made so that $20,000 will be accumulated at the end of 10 years. The interest rate on these investments is initially expected to be 4 percent compounded annually. After 4 years, the rate of interest is unexpectedly increased to 5 percent, so that payments for the remaining 6 years can be reduced. What amounts should be invested annually for the first 4 years and what sums for the last 6? 

Ans.: Final payment, $1547.

3.3. A firm wishes to set aside equal amounts at the end of each of 10 years, beginning at the end of the first year, in order to have $8000 in maintenance
funds available at the end of the seventh, eighth, ninth, and tenth years. What is the required annually payment if the money is invested and draws 6 percent compounded annually?

Ans.: $2655.

3.4. A home mortgage extends for 20 years at 8 percent interest compounded monthly. The payments are also made monthly. After how many months is half of the principal paid off?

Ans.: 164 months.

3.5. A lender offers a 1-year loan at what he calls 8 percent interest but requires the interest to be paid at the beginning rather than at the end of the year, as the usual practice is. To what interest rate computed in the conventional manner does this interest charge correspond?

Ans.: 8.7%.

3.6. A loan of $50,000 at 8 percent compounded annually is to be paid off in 25 years by uniform annual payments beginning at the end of the first year. These annual payments proceed on schedule until the end of the eighth year, when the borrower is unable to pay and misses the payment. He negotiates with the lender to increase the remaining 17 payments in such a way that the lender continues to receive 8 percent. What is the amount of the original and the final payments in the series?

Ans.: Final payments, $5197.44.

3.7. An $18,000 mortgage on which 8 percent interest is paid, compounded monthly, is to be paid off in 15 years in equal monthly installments. What is the total amount of interest paid during the life of this mortgage?

Ans.: $12,964.

3.8. What will be the future worth of a series of 15 annual $1000 payments if the nominal annual interest rate is 8 percent and the interest is compounded quarterly?

Ans.: $27,671.

3.9. A sum of sufficient magnitude is to be invested now so that starting 10 years from now an amount of $2000 per year can be paid in each of 8 succeeding years. The unexpended money remains invested at 8 percent compounded annually. How much must be allocated now?

Ans.: $5749.50.

3.10. A mortgage that was originally $20,000 is being paid off in regular quarterly payments of $500. The interest is 8 percent compounded quarterly. How much of the principal remains after 9 years, or 36 payments?

Ans.: $14,800.60.

3.11. A 20-year mortgage set up for uniform monthly payments with 6 percent interest compounded monthly is taken over by a new owner after 8 years. At that time $12,000 is still owed on the principal. What was the amount of the original loan?

Ans.: $16,345.

3.12. An investor buys common stock in a firm for $1000. At the end of the first year and every year thereafter, she receives a dividend of $100, which she immediately invests in a savings and loan institution that pays 5 percent interest compounded annually. At the end of the tenth year, just after receiving
her dividend, she sells the stock for $1200. What is the rate of interest (on an annual compounding basis) yielded by this investment program?

**Ans.**: 9.41%.

3.13. A sum of $20,000 is borrowed at an interest rate of 8 percent on the unpaid balance compounded semiannually. The loan is to be paid back with 10 equal payments in 20 years. The payments are to be made every 2 years, starting at the end of the second year. What is the amount of each biennial payment?

**Ans.**: $4291.

3.14. The packing in a cooling tower that cools condensing water for a power plant progressively deteriorates and results in gradually rising costs due to reduced plant efficiency. These costs are treated as lump sums at the end of the year, as shown in Fig. 3-6. The cost is zero for the first year and then increases $1000 per year until the packing is 16 years old, when replacement is mandatory. At a point 8 years into the life of the packing (just after the $7000 annual cost has been assessed), a decision is to be made on the plan for the next 8 years, i.e., whether to replace the packing or to continue with the existing packing. Money can be borrowed at 9 percent interest, compounded annually. What is the maximum amount that could be paid for the packing in order to justify its replacement?

**Ans.**: $44,279.

3.15. The anticipated taxes on a facility for its 10-year tax life decline in a straight-line fashion as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Tax, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>end of year 1</td>
<td>10,000</td>
</tr>
<tr>
<td>end of year 2</td>
<td>9,000</td>
</tr>
<tr>
<td>end of year 10</td>
<td>1,000</td>
</tr>
</tbody>
</table>

**FIGURE 3.6**
Increasing costs due to cooling-tower deterioration.
In an economic analysis of the facility the present worth of this series must be computed on the basis of 6 percent interest compounded annually. 

(a) Using a combination of available factors, determine a formula for the present worth of a declining series like this one.

(b) Using the formula from part (a), compute the present worth of the above series.

\[ \text{Ans.: (b) $43,999.} \]

3.16. Calculate the uniform annual profit on a processing plant for which the following data apply:

<table>
<thead>
<tr>
<th>Life</th>
<th>12 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>First cost</td>
<td>$280,000</td>
</tr>
<tr>
<td>Annual real estate tax and insurance</td>
<td>4% of first cost</td>
</tr>
<tr>
<td>Salvage value at end of 12 years</td>
<td>$50,000</td>
</tr>
<tr>
<td>Annual cost of raw materials, labor, and other supplies</td>
<td>$60,000</td>
</tr>
<tr>
<td>Annual income</td>
<td>$140,000</td>
</tr>
<tr>
<td>Maintenance costs, during first year</td>
<td>$0</td>
</tr>
<tr>
<td>At end of second year</td>
<td>$1000</td>
</tr>
<tr>
<td>At end of third year</td>
<td>$2000</td>
</tr>
<tr>
<td>At end of twelfth year</td>
<td>$11,000</td>
</tr>
</tbody>
</table>

The interest rate applicable is 6 percent compounded annually.

\[ \text{Ans.: $33,560.} \]

3.17. A $1000 bond was issued 5 years ago and will mature 5 years from now. The bond yields an interest rate of 5 percent, or $50 per year. The owner of the bond wishes to sell the bond, but since interest rates have increased, a prospective buyer wishes to earn a rate of 6 percent on his investment. What should the selling price be? Remember that the purchaser receives $50 per year, which is reinvested, and receives the $1000 face value at maturity. Interest is compounded annually.

\[ \text{Ans.: $957.88.} \]

3.18. Equation (3.8) relates the value of a bond \( P_R \) to the bond interest and current rate of interest by reflecting all values to a future worth. Develop an equation that reflects all values to a uniform semiannual worth and solve Example 3.9 with this equation.

3.19. Using a computer program, calculate tables of the price of a $1000 bond that will yield 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, and 11.0 percent interest when the interest rates on the bond are 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, and 10.0 percent. Compute the foregoing table for each of the following number of years to maturity: 2, 3, 4, 5, 6, 7, 8, 9, and 10. Interest is compounded semiannually.

3.20. A municipality must build a new electric generating plant and can choose between a steam or a hydro facility. The anticipated cost of the steam plant is $10 million. Comparative data for the two plants are
<table>
<thead>
<tr>
<th>Plant</th>
<th>Generating cost, including maintenance, per kWh</th>
<th>Equipment life, years</th>
<th>Salvage value, percentage of first cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam</td>
<td>$0.004</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Hydro</td>
<td>0.002</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

The expected annual consumption of power is 300,000 MWh. If money is borrowed at 5 percent interest compounded annually, what first cost of the hydro plant would make the two alternatives equally attractive investments?  
**Ans.:** $21,593,000.

3.21. A proposed investment consists of constructing a building, purchasing production machinery, and operating for 20 years. The expected life of the building is 20 years; its first cost is $250,000 with a salvage value of $50,000. Since the maximum life of the machinery is 12 years, it will be necessary to renew the machinery once during the 20 years. The first cost of the machinery is $132,000, and its salvage value is $132,000/(age, years). The annual income less the operating expense is expected to be $50,000. Annual interest is 6 percent compounded annually.

(a) When is the most favorable time to replace the machinery?

(b) Compute the present worth of the profit if the machinery is replaced at the time indicated by part (a).

**Ans.:** (b) $152,100.

3.22. Owners of a plant that manufactures edible oil are considering constructing a tank to store unrefined oil; this will permit buying raw oil at more favorable prices. The cost of the tank is $150,000, it has an expected life of 10 years and a salvage value at the end of its life of $20,000. The anticipated annual saving in oil cost is $25,000. What is the rate of return on the investment?

**Ans.:** 11.65%.

3.23. A new facility is expected to show zero profit during each of the first 5 years. For the 10 years thereafter the expected profit is to be $80,000 per year. If 12 percent interest (compounded annually) is desired as the return on the investment, what amount of investment is justified?

**Ans.:** $256,500.

3.24. The anticipated income from an investment is $40,000 per year for the first 5 years and $30,000 per year for the remaining 5 years of life. The desired rate of return on the investment is 12 percent. The salvage value at the end of 10 years is expected to be 20 percent of the first cost. Determine the first cost that will result in the 12 percent return.

**Ans.:** $219,700.

3.25. How many years will be required to double an investment if it draws interest at a rate of 8 percent, compounded semiannually?

3.26. You just received a notice of an insurance premium of $45 per month starting October 1 but were also offered the option of making one annual payment on October 1. If you want 10 percent annual rate of interest, what annual payment would you be willing to make?

**Ans.:** $516.12.
3.27. A sum of $10,000 is invested and draws interest at a rate of 8 percent, compounded annually. Starting at the end of the first year and each year thereafter $1000 is withdrawn. For how many years can this plan continue until the money is exhausted?

Ans.: 21 years.

3.28. A firm borrows $200,000 at 9 percent nominal interest, compounded monthly and is to repay the loan in 12 years with regular monthly payments of $2276.06. The firm has the option of paying off in advance, and after the sixth year makes an additional $50,000 payment. If it continues the $2276.06 payments, how many additional months are required to pay off the loan?

Ans.: 39 months

3.29. A car rental agency which leases cars to another firm buys cars for $9,000 and sells them for $6,000 two years later. It charges a monthly rate the second year of rental of 80 percent of that of the first year. The agency seeks 1 percent per month return. What are the monthly rates each year?

Ans.: 2nd year, $177.74

3.30. A 20-year loan is to be paid off by monthly payments of $M$. The nominal annual interest rate is $i$. Develop a closed-form expression for the unpaid balance at year $n$.

3.31. A firm has capital on hand and is considering an investment in a plant that is expected to show a net annual return (income less expense) of $80,000 per year. The life of the facility is 10 years, and the salvage value at the end of that time is 20 percent of the first cost. If the firm wishes a 12 percent return on its investment, how much can it justify as the first cost?

Ans.: $483,131.

3.32. The first cost of an investment is $600,000 borrowed at 11 percent interest compounded semiannually. The expected income (less operating expense) for every 6-month period is $60,000. If there is no salvage value, how long must the plant operate in order to pay off the investment?

Ans.: 7 1/2 years

3.33. A 15-year mortgage of $40,000 at 10 percent interest compounded monthly is to be paid off with monthly payments. How much total interest will be paid during the first two years?

Ans.: $7,763.57

3.34. A sum of $1,000 is invested and draws interest at the rate of 8 percent compounded annually. At the end of the first year and each year thereafter $50 is withdrawn from the invested amount. How much money is still available in the investment after the 20th annual withdrawal?

Ans.: $2372.96

3.35. In a certain financing arrangement the sum of $100,000 is loaned at 12 percent compounded monthly as though it were to be paid off in 25 years. At the end of 5 years the agreement calls for the borrower to pay back the unpaid balance at that time. What is the unpaid balance after 5 years?

Ans.: $95,653.

3.36. The expected annual income from a new facility that is under consideration is $120,000, and the anticipated annual operating expenditures are $60,000. The salvage value at the end of the expected life of 12 years will be 20
percent of the first cost. What first cost would result in a rate of return of 15 percent?

\textbf{Ans.:} $337,868.

3.37. A processing plant has a first cost of $600,000 and an expected life of 15 years with no salvage value. Money is borrowed at 8 percent compounded annually, and the first cost is paid off with 15 equal annual payments. The expected annual income is $200,000, and annual operating expenses are $40,000. Corporation income tax is 50 percent of the profits before taxes, and the SYD method of depreciation is applicable on the tax life of the facility, which is 12 years with no salvage value. Compute the income tax for (a) the first year and (b) the second year.

\textbf{Ans.:} (a) $9846; (b) $14,576.

3.38. A client who is constructing a warehouse instructs the contractor to omit insulation. The client explains that he will operate the building for several months and then install the insulation as a repair, so that he can deduct the expense from income tax at the end of the first year rather than spread it as straight-line depreciation over the 8-year tax life of the warehouse. The contractor points out that a later installation will cost more than the $20,000 cost of installing the insulation with the original construction. How much could the client afford to pay for the later installation for equal profit if he plans on a 15 percent return on his investment and corporation income taxes are 50 percent?

\textbf{Ans.:} $25,461.

3.39. A $200,000 facility has an 8-year tax life, and the firm expects a 12 percent return on its investment and pays 50 percent corporation income tax on profits. The firm is comparing the relative advantage of the SYD and straight-line methods of depreciation. If the taxes computed by the two methods are expressed as uniform annual amounts, what is the advantage of the SYD method?

\textbf{Ans.:} $1630.

3.40. A firm borrows $250,000 for a facility that it will pay off in 10 equal installments at 12 percent interest, compounded annually. In computing income tax the firm can deduct the \textit{actual} interest paid during the year. What is the actual interest paid the second year?

\textbf{Ans.:} $28,290.

3.41. An investor pays $80,000 for a building and expects to sell it for twice that amount at the end of eight years. He can depreciate the building on a straight-line basis during the eight years, or he can charge off no depreciation at all. On the capital gains of the sale at the end of eight years, which is $160,000 – ($80,000 – depreciation) he pays half the income tax that he does on regular income. State which is the most profitable depreciation plan and give all the reasons why it is most profitable.

3.42. Regular payments of $1400 are to be made annually, starting at the end of the first year. These amounts will be invested at 6 percent compounded continuously. How many years will be needed for the payments plus interest to accumulate to $24,000?

\textbf{Ans.:} 12 years.
3.43. An investment of $300,000 yields an annual profit of $86,000 that is spread uniformly over the year and is reinvested immediately (thus continuously compounded). The life is 6 years, and there is no salvage value. What is the rate of return on the investment?

Ans.: 20%.

ADDITIONAL READINGS